

$$T(\alpha x + \beta y) = \alpha T(x) + \beta T(y) \quad \alpha, \beta \in F, \quad x, y \in V$$

$$1) a) \quad T(x, y) = (x+y, x-y) \quad T: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \quad T: V \rightarrow V$$

$$x = (x_1, y_1) \quad y = (x_2, y_2)$$

$$T(\alpha x + \beta y) = T[\alpha(x_1, y_1) + \beta(x_2, y_2)]$$

$$= T[\alpha x_1 + \beta x_2, \alpha y_1 + \beta y_2]$$

$$= (\alpha x_1 + \beta x_2 + \alpha y_1 + \beta y_2, \alpha x_1 + \beta x_2 - \alpha y_1 - \beta y_2)$$

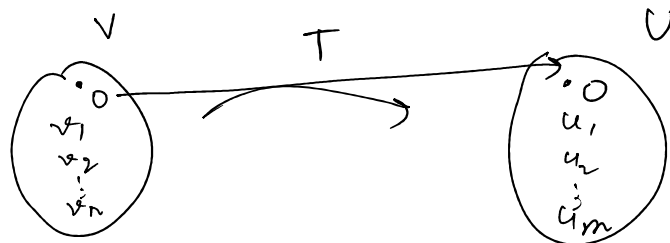
$$\alpha T(x) + \beta T(y) = \alpha T(x_1, y_1) + \beta T(x_2, y_2)$$

$$\begin{aligned} &= \alpha(x_1+y_1, x_1-y_1) + \beta(x_2+y_2, x_2-y_2) \\ &= (\alpha x_1 + \alpha y_1 + \beta x_2 + \beta y_2, \alpha x_1 - \alpha y_1 + \beta x_2 - \beta y_2) \\ &\rightarrow \alpha \begin{bmatrix} x_1+y_1 \\ x_1-y_1 \end{bmatrix} + \beta \begin{bmatrix} x_2+y_2 \\ x_2-y_2 \end{bmatrix} \end{aligned}$$

Null space of  $T = N(T) = \text{kernel of } T$

$$= \{ x \in V : Tx = 0 \}$$

$$\text{Range space } R(T) = \{ Tx : x \in V \}$$



$$N(T) = \{ (x, y) \in \mathbb{R}^2 : T(x, y) = (0, 0) \}$$

$$= \{ \quad : (x+y, x-y) = (0, 0) \}$$

$$x+y=0, \quad x-y=0$$

$$x=0, y=0$$

$$= \{ (0, 0) \}$$

$$\dim N(T) = 0$$

$$= \{ \quad : \quad \in \mathbb{R}^2 \}$$

$$\dim N(T) = 0$$

$$R(T) = \{ T(x, y) : (x, y) \in \mathbb{R}^2 \}$$

$$\text{Basis of } \mathbb{R}^2 = \{ (1, 0), (0, 1) \}$$

$$T(1, 0) = \underline{(1, 1)} \quad T(0, 1) = \underline{(1, -1)}$$

$$\text{Span of } R(T) = \{ (1, 1), (1, -1) \}$$

$$\rightarrow \text{Basis of } R(T)$$

$$\dim R(T) = 2$$

$$\dim V = \dim N(T) + \dim R(T)$$

$$2 = 0 + 2$$

Rank Nullity thm.